1970-04

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http://hdl.handle.net/10945/15046

INVESTIGATION OF THE ACCURACY OF USING STEADY-STATE RESULTS TO APPROXIMATE ACTUAL SYSTEM AVAILABILITY

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THESIS

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STEADY-STATE RESULTS TO APPROXIMATE
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by

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April 1970

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
April 1970

ABSTRACT

Values of system availability are computed for a system whose failure density is exponential (λ) and whose repair density is special Erlang (μ,α). The system is modeled as an alternating renewal process. The Laplace transform of the availability function is developed and inverted (by using a numerical procedure) to obtain the values of the system availability.

Corresponding values of the long-run average system availability and the average availability over the first mission time are also computed. Comparisons are made which establish that using the long-run average value to approximate the true availability over the first mission time is a conservative practice.

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I. INTRODUCTION

A. THE SYSTEM

Most physical systems people encounter are of the type which operate for a time, fail, are repaired and put back into operation. The system then may be said to have two states. Truely, each of these states may be the union of numerous sub-states, but the two major states prevail.

Some usual terms applied to the first state are 'operational', 'available' or 'up'. This is the state or condition of the system when it is capable of accomplishing the task for which it was designed and built. The second state, the compliment of the first, is the less desirable state and is usually termed 'failed', 'nonoperational', 'unavailable' or 'down'.

Hereinafter the system states will be referred to as 'up' or 'down'.

B. DEFINITION OF AVAILABILITY

A good deal of effort is being expended by the U.S. Government and by private industry to obtain measures of certain quality characteristics applicable to such systems as is described in A. Hosford [2] has discussed several of these characteristics.

One such quality characteristic that is of primary concern is system availability. The 'system availability' or simply 'availability' is the probability that the

system is up at any time $t \geq 0$, given it was up at time zero.

The availability function, $\eta(t)$, is a real valued function which gives this probability for any time $t \geq 0$.

C. METHOD CURRENTLY PRACTICED TO DETERMINE AVAILABILITY

In reliability literature, availability is often defined differently as:

$$A = \frac{MTTF}{MTTF + MRT}$$

where MTTF is Mean-Time-To-Failure and MRT is Mean-Repair-Time. This definition provides the only method currently in wide spread use for determining availability.

Note: This quantity has the variable name ZBAR in the computer program and output (Appendices A and C).

Cox [1] has shown that regardless of the distributions of uptimes and downtimes

$$\lim_{t \to \infty} \eta(t) = A.$$

That is, A is actually the steady-state system availability.

Similarly, it may be shown that the long-run average availability

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t \eta(x)dx = A.$$

Jacobson [3] describes a statistical procedure for obtaining a confidence interval estimate for A.

D. OBJECTIVES OF THIS RESEARCH

The objectives of this research were the following:

1. To compute exact values of the availability function for a system with given distributions of uptimes and downtimes.

2. To compare true availability of a system with that obtained by methods currently in practice.

II. SUMMARY AND CONCLUSIONS

A physical system having an 'up' state and a 'down' state and with the ability to transit between the two states was modeled as an alternating renewal process for the purpose of finding the system's availability function, $\eta(t)$. An explicit general expression for $\eta(t)$ was not found because of the complex structure of the model equations under the assumed distributions of uptimes and downtimes. However, $\eta^*(s)$, the Laplace transform of $\eta(t)$, was developed from the assumed probability density functions and the renewal density functions of the model.

A computer program was written to use a numerical procedure to invert the Laplace transform for specific values of the parameters and of time, the independent variable.

Upon consideration of the results and some sample problems (see Appendices A and B) the following conclusions were made.

1. Under the fairly realistic assumptions of the model, exact values of $\eta(t)$ can be and were computed.

2. Use of long-run average availability to approximate the true availability over the first mission time, as is currently being done in practice, is a conservative procedure.

3. The amount by which the long-run average availability underestimates the true availability during the first mission time is dependent upon the long-run value, i.e., the lower the long-run value, the more it underestimates the true availability.

III. THE MODEL

A. DESCRIPTION

The system was modeled as an alternating renewal process with two states. That is, the new system was assumed to start operating in the up state. The time spent in this state before the first failure, whether in use or just available for use, was a random variable U_1. Upon failure the system entered the down state. The time spent down before the first repair, whether awaiting repairs or actually being repaired, was a random variable D_1. The length of the first cycle was the random time $U_1 + D_1$.

It was assumed the system kept alternating between up and down and each repair returned it to 'like new' condition so that the uptimes U_i, i=1,2,..., were taken to be independent, identically distributed random variables. They were assumed to be exponentially distributed with parameter $\lambda > 0$ and probability density function

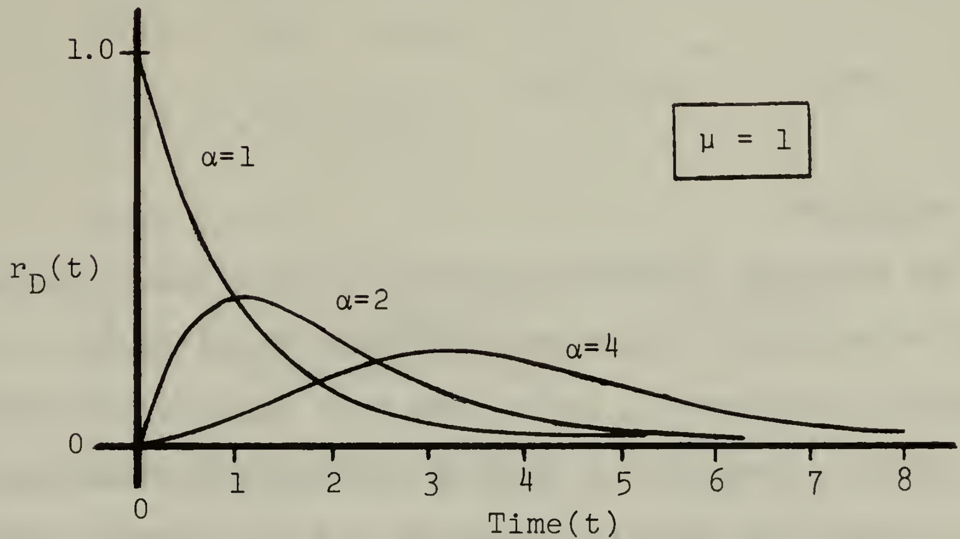
$$f_U(t) = \begin{cases} 0 & , \ t < 0 \\ \lambda e^{-\lambda t} & , \ 0 \leq t \ . \end{cases}$$

Note: All time units in this thesis are mission times.

The downtimes D_i,i=1,2,..., were also assumed to be independent, identically distributed random variables and independent of the uptimes. They were assumed to have a

special Erlang distribution with parameters $\mu > 0$ and $\alpha = 1, 2, \ldots,$ and with probability density function.

$$r_D(t) = \begin{cases} 0 & , \quad t < 0 \\ \dfrac{\mu^\alpha}{(\alpha-1)!}\, t^{\alpha-1}\, e^{-\mu t} & , \quad 0 \le t \end{cases}$$



The Special Erlang Density Function

Weiss [5], through a simple argument, has shown the renewal density functions for such a renewal process to be

$$W_0(t) = f_U(t) + \int_0^t W_1(\tau)\, f_U(t-\tau)\, d\tau$$

$$W_1(t) = \int_0^t W_0(\tau)\, r_D(t-\tau)\, d\tau$$

where 0 and 1 are subscripts for 'up' and 'down' respectively and $W_j(t)$ is the renewal density for the event "end of stay in state j". Weiss has also given the availability function as:

$$\eta(t) = R(t) + \int_0^t W_1(\tau)\, R(t-\tau)\, d\tau$$

where R(t) is the reliability function, i.e.,

$$R(t) = \int_t^\infty f_U(x)\, dx.$$

B. SOLUTION

No straight forward solution of these equations for $\eta(t)$ in explicit closed form has been found except in the special case where f_U and r_D are both exponential density functions. However, by finding the Laplace transforms of the probability density functions and the renewal density functions the Laplace transform of the availability function was found.

1. Laplace Transforms

The Laplace transform of a continuous or sectionally continuous function g(t), t in $[0,\infty)$, is

$$g^*(s) = \int_0^\infty e^{-st}\, g(t)dt \;;\quad s \text{ is a complex constant}$$

whose real part is large enough that the integral converges.

The Laplace transform of $f_U(t)$ is

$$f_U^*(s) = \int_0^\infty e^{-st}\, f_U(t)dt = \int_0^\infty e^{-st}\,\lambda\, e^{-\lambda t}\, dt$$

$$= \frac{\lambda}{\lambda+s} \int_0^\infty (\lambda+s)\, e^{-(\lambda+s)t}\, dt = \frac{\lambda}{\lambda+s} .$$

Similarly, $r_D^*(s)$ may be shown to equal $\left(\frac{\mu}{\mu+s}\right)^\alpha$.

The Laplace transforms of the renewal densities are:

$$w_0^*(s) = f_U^*(s) + w_1^*(s)\, f_U^*(s)$$

$$w_1^*(s) = w_0^*(s)\, r_D^*(s) .$$

These imply

$$w_0^*(s) = \frac{\lambda(\mu+s)^\alpha}{(\lambda+s)(\mu+s)^\alpha - \lambda\mu^\alpha} .$$

Noting that in this case

$$\eta(t) = \frac{w_0(t)}{\lambda} ,$$

the Laplace transform of the availability function was found to be

$$\eta^*(s) = \frac{(\mu+s)^\alpha}{(\lambda+s)(\mu+s)^\alpha - \lambda\mu^\alpha} .$$

2. The Inverse Transform

The desired solution for the availability function is the inverse transform of $\eta^*(s)$. As before, no method was found to obtain an explicit general expression for $\eta(t)$.

Therefore a numerical inversion procedure was used to compute values of $\eta(t)$ for particular values of the parameters λ, μ, and α and for values of $t=.05$, .10, .15, ..., 1.0 mission times.

Several values were chosen for the parameters λ, μ and α over a range that was judged to be of practical interest. Those values and the corresponding values of $\eta(t)$ are shown in Appendix A.

IV. DESCRIPTION OF THE COMPUTER PROGRAM

The computations were done by a computer program written in FORTRAN IV programming language and run on the IBM system/360, model 67 computer at the Naval Postgraduate School (see program listing, Appendix C). The program listing contains enough explanatory comments to enable the reader familiar with FORTRAN to follow the procedure.

The following are the major steps:

1. The polynomial in the denominator of $\eta^*(s)$ was expanded to the form $a_0 + a_1 s + a_2 s^2 + \ldots + s^{\alpha+1}$. That is,

$$(\lambda+s)(\mu+s)^\alpha - \lambda\mu^\alpha = \sum_{j=1}^{\alpha} \binom{\alpha}{j} \mu^{\alpha-j} \left(\lambda + \frac{j\mu}{\alpha+1-j}\right) s^j + s^{\alpha+1} .$$

Note that a_0 equals zero.

2. A computer library subroutine which employs the Newton-Raphson successive approximations technique was used to find the roots of this polynomial.

3. After the roots were checked to see that they were all different the following formula was used in computing the values of $\eta(t)$. Often called Heaviside's expansion formula [4] it is an application of the residue theorem from theory of complex variables.

Heaviside's Expansion Formula: let P(s) and Q(s) be polynomials where P(s) has degree less than that of Q(s). If Q(s) has distinct roots A_k, k=1,2,...,n, then the inverse Laplace transform of $\frac{P(s)}{Q(s)}$ is

$$L^{-1}\left(\frac{P(s)}{Q(s)}\right) = \sum_{k=1}^{n} \frac{P(A_k)}{Q'(A_k)} e^{A_k t} \quad .$$

By application of this formula

$$L^{-1}(\eta^*(s)) = \eta(t) = \sum_{k=1}^{\alpha+1} \frac{\mu+A_k}{\mu+A_k+\alpha(\lambda+A_k)} e^{A_k t} \quad .$$

APPENDIX A

RESULTS

This appendix contains tabulated values of the availability function for the indicated sets of parameters and the indicated time in mission-time units.

The values of PZERO given are such that PZERO equals the probability of a repair in less than one mission time, i.e.,

$$\int_0^1 \frac{\mu^\alpha}{(\alpha-1)!} t^{\alpha-1} e^{-\mu t} \, dt = \text{PZERO} .$$

Z1 is the average availability over the first mission time.

$$Z1 = \frac{1}{20} \left(\frac{\eta(0)+\eta(1)}{2} + \sum_{i=1}^{19} \eta(.05i) \right)$$

ZBAR is the asymptotic value of $\eta(t)$ for large t. ZBAR is also the long-run average availability.

$$\text{ZBAR} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTR}} = \frac{1/\lambda}{1/\lambda + \alpha/\mu} = \frac{\mu}{\mu + \alpha\lambda}$$

Example: For a system whose uptimes have the exponential ($\lambda = .005$) distribution and whose downtimes have the Erlang ($\mu = 2.03$, $\alpha = 2$) distribution, the true system availability at a half mission time is $\eta(.5) = .99777$.

The true average availability over the first mission time is Z1 = .99792 and the long-run average availability is ZBAR = .99510.

ALPHA= 2.0 PZERO= 0.60 MU= 2.03

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99751	.99502	.99007	.98514	.98023
0.10	.99950	.99901	.99504	.99011	.98032	.97063	.96103
0.15	.99926	.99852	.99263	.98531	.97084	.95658	.94254
0.20	.99902	.99805	.99027	.98064	.96167	.94307	.92484
0.25	.99879	.99759	.98799	.97614	.95286	.93016	.90802
0.30	.99857	.99714	.98580	.97180	.94444	.91788	.89210
0.35	.99836	.99671	.98369	.96766	.93642	.90626	.87712
0.40	.99815	.99631	.98167	.96370	.92882	.89530	.86308
0.45	.99795	.99591	.97975	.95995	.92164	.88501	.84997
0.50	.99777	.99554	.97793	.95639	.91489	.87538	.83778
0.55	.99759	.99519	.97620	.95303	.90854	.86640	.82647
0.60	.99742	.99485	.97456	.94987	.90260	.85804	.81602
0.65	.99726	.99454	.97302	.94689	.89706	.85029	.80639
0.70	.99711	.99424	.97157	.94410	.89189	.84312	.79754
0.75	.99697	.99396	.97021	.94149	.88709	.83650	.78943
0.80	.99684	.99369	.96894	.93905	.88264	.83040	.78201
0.85	.99671	.99344	.96774	.93678	.87851	.82479	.77524
0.90	.99660	.99321	.96663	.93466	.87470	.81965	.76908
0.95	.99649	.99299	.96559	.93270	.87119	.81494	.76349
1.00	.99639	.99279	.96462	.93087	.86795	.81064	.75842
Z1	.99792	.99584	.97945	.95954	.92151	.88574	.85208
ZBAR	.99510	.99024	.95305	.91031	.83539	.77186	.71731

ALPHA= 2.0 PZERO= 0.70 MU= 2.45

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99751	.99502	.99007	.98515	.98025
0.10	.99950	.99901	.99506	.99014	.98037	.97071	.96114
0.15	.99926	.99853	.99267	.98539	.97100	.95682	.94286
0.20	.99903	.99806	.99036	.98082	.96202	.94360	.92553
0.25	.99881	.99762	.98815	.97646	.95349	.93109	.90925
0.30	.99860	.99719	.98605	.97231	.94544	.91935	.89404
0.35	.99839	.99679	.98406	.96840	.93788	.90840	.87992
0.40	.99820	.99641	.98219	.96472	.93081	.89822	.86689
0.45	.99802	.99605	.98043	.96128	.92424	.88881	.85492
0.50	.99785	.99571	.97878	.95807	.91816	.88016	.84397
0.55	.99770	.99540	.97725	.95510	.91254	.87222	.83400
0.60	.99755	.99511	.97582	.95234	.90738	.86497	.82495
0.65	.99741	.99484	.97450	.94979	.90264	.85836	.81676
0.70	.99729	.99458	.97328	.94745	.89831	.85236	.80938
0.75	.99717	.99435	.97216	.94529	.89436	.84693	.80275
0.80	.99706	.99414	.97112	.94331	.89076	.84203	.79681
0.85	.99696	.99394	.97017	.94151	.88750	.83761	.79150
0.90	.99687	.99376	.96930	.93985	.88454	.83364	.78678
0.95	.99679	.99359	.96850	.93835	.88186	.83008	.78257
1.00	.99671	.99343	.96777	.93698	.87945	.82689	.77885
Z1	.99803	.99606	.98056	.96170	.92566	.89170	.85968
ZBAR	.99593	.99190	.96078	.92453	.85965	.80328	.75385

ALPHA= 2.0   PZERO= 0.80   MU= 3.00

TIME	LAMBDA .005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99751	.99503	.99008	.98516	.98027
0.10	.99951	.99901	.99508	.99018	.98045	.97083	.96130
0.15	.99927	.99854	.99273	.98552	.97125	.95719	.94334
0.20	.99905	.99809	.99050	.98109	.96255	.94437	.92656
0.25	.99883	.99767	.98839	.97692	.95441	.93245	.91103
0.30	.99863	.99727	.98642	.97304	.94687	.92146	.89680
0.35	.99845	.99690	.98459	.96944	.93992	.91141	.88386
0.40	.99827	.99655	.98290	.96613	.93357	.90226	.87216
0.45	.99812	.99624	.98135	.96310	.92779	.89400	.86166
0.50	.99797	.99595	.97993	.96034	.92256	.88657	.85227
0.55	.99784	.99568	.97863	.95783	.91784	.87991	.84393
0.60	.99772	.99544	.97746	.95556	.91360	.87398	.83655
0.65	.99761	.99522	.97640	.95352	.90981	.86871	.83004
0.70	.99751	.99502	.97544	.95168	.90643	.86404	.82433
0.75	.99742	.99484	.97458	.95003	.90342	.85993	.81933
0.80	.99734	.99468	.97381	.94856	.90076	.85632	.81498
0.85	.99726	.99454	.97311	.94724	.89840	.85315	.81120
0.90	.99720	.99441	.97249	.94607	.89632	.85037	.80792
0.95	.99714	.99429	.97194	.94503	.89449	.84796	.80509
1.00	.99709	.99419	.97145	.94411	.89288	.84586	.80266
Z1	.99817	.99635	.98195	.96442	.93085	.89915	.86920
ZBAR	.99668	.99338	.96774	.93750	.88235	.83333	.78947

ALPHA= 2.0   PZERO= 0.90   MU= 3.90

TIME	LAMBDA .005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99752	.99504	.99011	.98520	.98031
0.10	.99951	.99902	.99512	.99026	.98061	.97106	.96161
0.15	.99928	.99857	.99285	.98575	.97171	.95788	.94426
0.20	.99907	.99814	.99074	.98158	.96352	.94582	.92846
0.25	.99888	.99775	.98881	.97777	.95608	.93492	.91428
0.30	.99870	.99740	.98707	.97433	.94940	.92520	.90170
0.35	.99854	.99708	.98550	.97125	.94346	.91661	.89066
0.40	.99840	.99680	.98410	.96851	.93823	.90909	.88106
0.45	.99827	.99654	.98287	.96610	.93364	.90255	.87276
0.50	.99816	.99632	.98178	.96399	.92965	.89689	.86565
0.55	.99806	.99613	.98083	.96215	.92619	.89204	.85958
0.60	.99797	.99595	.98000	.96054	.92321	.88789	.85444
0.65	.99790	.99581	.97928	.95916	.92066	.88435	.85011
0.70	.99783	.99568	.97865	.95796	.91847	.88136	.84647
0.75	.99778	.99556	.97811	.95694	.91661	.87884	.84343
0.80	.99773	.99547	.97765	.95605	.91503	.87672	.84090
0.85	.99769	.99538	.97725	.95530	.91370	.87494	.83881
0.90	.99765	.99531	.97691	.95466	.91257	.87346	.83708
0.95	.99762	.99525	.97662	.95412	.91162	.87223	.83566
1.00	.99759	.99520	.97637	.95365	.91083	.87121	.83450
Z1	.99838	.99676	.98399	.96841	.93850	.91013	.88323
ZBAR	.99744	.99490	.97500	.95122	.90698	.86667	.82979

ALPHA= 2.0 PZERO= 0.95 MU= 4.80

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99752	.99505	.99013	.98524	.98037
0.10	.99952	.99903	.99516	.99035	.98080	.97134	.96198
0.15	.99930	.99859	.99298	.98602	.97225	.95868	.94531
0.20	.99910	.99820	.99102	.98212	.96460	.94741	.93057
0.25	.99892	.99784	.98927	.97868	.95787	.93757	.91776
0.30	.99877	.99753	.98775	.97567	.95204	.92910	.90681
0.35	.99863	.99727	.98643	.97308	.94705	.92189	.89755
0.40	.99852	.99704	.98529	.97086	.94282	.91581	.88981
0.45	.99842	.99684	.98433	.96899	.93925	.91074	.88340
0.50	.99833	.99667	.98352	.96741	.93628	.90654	.87813
0.55	.99826	.99653	.98283	.96608	.93381	.90308	.87383
0.60	.99820	.99641	.98226	.96498	.93177	.90025	.87035
0.65	.99815	.99631	.98178	.96407	.93009	.89795	.86754
0.70	.99811	.99623	.98138	.96331	.92872	.89609	.86529
0.75	.99808	.99616	.98106	.96269	.92760	.89458	.86349
0.80	.99805	.99611	.98079	.96218	.92669	.89338	.86207
0.85	.99803	.99606	.98056	.96176	.92596	.89242	.86095
0.90	.99801	.99602	.98038	.96142	.92537	.89165	.86007
0.95	.99799	.99599	.98023	.96114	.92490	.89104	.85938
1.00	.99798	.99596	.98011	.96092	.92452	.89056	.85885
Z1	.99856	.99712	.98573	.97182	.94501	.91950	.89521
ZBAR	.99792	.99585	.97959	.96000	.92308	.88889	.85714

ALPHA= 3.0 PZERO= 0.60 MU= 3.11

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99502	.99006	.98022	.97048	.96083
0.15	.99925	.99851	.99255	.98516	.97054	.95614	.94195
0.20	.99901	.99802	.99012	.98034	.96106	.94217	.92365
0.25	.99877	.99753	.98773	.97562	.95184	.92865	.90603
0.30	.99853	.99706	.98541	.97103	.94291	.91563	.88916
0.35	.99830	.99661	.98315	.96659	.93434	.90320	.87313
0.40	.99808	.99616	.98098	.96233	.92615	.89138	.85799
0.45	.99787	.99574	.97889	.95826	.91837	.88023	.84379
0.50	.99766	.99534	.97691	.95439	.91102	.86978	.83055
0.55	.99747	.99495	.97503	.95074	.90413	.86002	.81828
0.60	.99729	.99459	.97325	.94730	.89768	.85097	.80698
0.65	.99712	.99424	.97158	.94408	.89170	.84262	.79664
0.70	.99695	.99392	.97002	.94108	.88616	.83495	.78721
0.75	.99680	.99362	.96857	.93830	.88106	.82795	.77868
0.80	.99666	.99334	.96722	.93572	.87638	.82159	.77099
0.85	.99653	.99308	.96597	.93334	.87210	.81583	.76410
0.90	.99641	.99284	.96481	.93116	.86822	.81064	.75796
0.95	.99630	.99261	.96375	.92917	.86469	.80599	.75251
1.00	.99619	.99241	.96278	.92735	.86151	.80183	.74770
Z1	.99782	.99564	.97849	.95767	.91797	.88071	.84572
ZBAR	.99520	.99045	.95399	.91202	.83827	.77556	.72158

ALPHA= 3.0 PZERO= 0.70 MU= 3.62

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98512	.98020
0.10	.99950	.99900	.99502	.99007	.98023	.97049	.96085
0.15	.99925	.99851	.99256	.98518	.97059	.95621	.94205
0.20	.99901	.99802	.99015	.98040	.96120	.94237	.92392
0.25	.99877	.99755	.98780	.97576	.95212	.92907	.90658
0.30	.99854	.99709	.98554	.97129	.94343	.91640	.89016
0.35	.99832	.99665	.98336	.96702	.93517	.90443	.87474
0.40	.99811	.99623	.98130	.96296	.92739	.89321	.86038
0.45	.99791	.99583	.97934	.95915	.92011	.88279	.84713
0.50	.99773	.99546	.97751	.95559	.91335	.87318	.83498
0.55	.99755	.99511	.97581	.95227	.90711	.86438	.82394
0.60	.99739	.99478	.97422	.94921	.90140	.85637	.81397
0.65	.99724	.99448	.97277	.94640	.89619	.84913	.80503
0.70	.99710	.99420	.97143	.94384	.89147	.84263	.79708
0.75	.99697	.99395	.97020	.94150	.88721	.83682	.79004
0.80	.99685	.99372	.96909	.93939	.88340	.83167	.78386
0.85	.99675	.99351	.96808	.93748	.87999	.82711	.77846
0.90	.99665	.99332	.96717	.93577	.87696	.82311	.77377
0.95	.99656	.99314	.96635	.93423	.87429	.81962	.76973
1.00	.99649	.99299	.96562	.93287	.87193	.81658	.76627
Z1	.99791	.99583	.97940	.95945	.92138	.88562	.85200
ZBAR	.99587	.99178	.96021	.92347	.85782	.80088	.75104

ALPHA= 3.0 PZERO= 0.80 MU= 4.30

TIME	LAMBDA .005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98512	.98021
0.10	.99950	.99900	.99503	.99008	.98025	.97052	.96089
0.15	.99926	.99851	.99259	.98523	.97067	.95634	.94221
0.20	.99902	.99803	.99021	.98052	.96142	.94270	.92436
0.25	.99879	.99757	.98792	.97600	.95259	.92976	.90750
0.30	.99857	.99713	.98575	.97171	.94426	.91762	.89177
0.35	.99836	.99672	.98370	.96768	.93648	.90636	.87728
0.40	.99816	.99633	.98179	.96394	.92930	.89603	.86407
0.45	.99798	.99597	.98002	.96049	.92274	.88665	.85215
0.50	.99782	.99564	.97841	.95735	.91678	.87820	.84151
0.55	.99766	.99534	.97694	.95450	.91143	.87068	.83210
0.60	.99753	.99506	.97561	.95193	.90666	.86402	.82385
0.65	.99740	.99482	.97441	.94964	.90243	.85818	.81668
0.70	.99729	.99460	.97335	.94760	.89872	.85309	.81050
0.75	.99719	.99440	.97240	.94581	.89547	.84869	.80522
0.80	.99711	.99422	.97157	.94423	.89264	.84491	.80074
0.85	.99703	.99407	.97083	.94284	.89020	.84169	.79697
0.90	.99696	.99393	.97019	.94164	.88810	.83896	.79383
0.95	.99690	.99381	.96963	.94060	.88631	.83666	.79122
1.00	.99685	.99371	.96914	.93970	.88479	.83474	.78908
Z1	.99803	.99608	.98062	.96183	.92595	.89218	.86038
ZBAR	.99652	.99307	.96629	.93478	.87755	.82692	.78182

ALPHA= 3.0 PZERO= 0.90 MU= 5.35

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99502	.99006	.98512	.98021
0.10	.99950	.99901	.99504	.99010	.98029	.97058	.96097
0.15	.99926	.99852	.99263	.98531	.97084	.95659	.94255
0.20	.99903	.99806	.99032	.98074	.96187	.94337	.92524
0.25	.99881	.99762	.98815	.97646	.95349	.93110	.90927
0.30	.99861	.99721	.98614	.97250	.94581	.91992	.89479
0.35	.99842	.99684	.98431	.96890	.93887	.90988	.88189
0.40	.99825	.99650	.98266	.96566	.93268	.90100	.87055
0.45	.99810	.99620	.98119	.96280	.92724	.89324	.86074
0.50	.99797	.99594	.97990	.96029	.92251	.88656	.85236
0.55	.99785	.99571	.97877	.95810	.91843	.88087	.84528
0.60	.99775	.99551	.97779	.95623	.91496	.87606	.83938
0.65	.99766	.99533	.97695	.95462	.91203	.87205	.83450
0.70	.99759	.99518	.97623	.95326	.90957	.86873	.83052
0.75	.99752	.99506	.97563	.95211	.90753	.86601	.82731
0.80	.99747	.99495	.97512	.95115	.90585	.86379	.82474
0.85	.99742	.99486	.97469	.95036	.90447	.86201	.82271
0.90	.99739	.99478	.97433	.94970	.90334	.86059	.82112
0.95	.99735	.99472	.97403	.94915	.90244	.85947	.81990
1.00	.99733	.99466	.97379	.94871	.90171	.85858	.81896
Z1	.99822	.99644	.98241	.96534	.93266	.90181	.87268
ZBAR	.99720	.99442	.97273	.94690	.89916	.85600	.81679

ALPHA= 3.0 PZERO= 0.95 MU= 6.30

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99751	.99502	.99006	.98513	.98022
0.10	.99950	.99901	.99505	.99012	.98034	.97066	.96107
0.15	.99927	.99853	.99268	.98541	.97105	.95689	.94295
0.20	.99904	.99808	.99045	.98100	.96238	.94413	.92624
0.25	.99883	.99767	.98841	.97696	.95448	.93257	.91119
0.30	.99865	.99730	.98656	.97333	.94744	.92233	.89796
0.35	.99848	.99697	.98493	.97013	.94129	.91345	.88656
0.40	.99834	.99668	.98352	.96736	.93600	.90587	.87692
0.45	.99821	.99643	.98230	.96500	.93153	.89952	.86891
0.50	.99811	.99622	.98127	.96300	.92779	.89426	.86235
0.55	.99802	.99604	.98041	.96134	.92470	.88997	.85705
0.60	.99794	.99589	.97969	.95997	.92219	.88652	.85284
0.65	.99788	.99577	.97910	.95884	.92015	.88377	.84954
0.70	.99783	.99567	.97862	.95793	.91853	.88160	.84698
0.75	.99779	.99559	.97823	.95720	.91724	.87991	.84502
0.80	.99776	.99552	.97792	.95661	.91623	.87861	.84354
0.85	.99773	.99547	.97766	.95615	.91544	.87762	.84245
0.90	.99771	.99542	.97746	.95578	.91484	.87688	.84164
0.95	.99769	.99539	.97730	.95549	.91437	.87632	.84106
1.00	.99768	.99536	.97718	.95527	.91402	.87591	.84065
Z1	.99837	.99674	.98388	.96821	.93815	.90970	.88274
ZBAR	.99762	.99526	.97674	.95455	.91304	.87500	.84000

ALPHA= 4.0 PZERO= 0.60 MU= 4.20

TIME	LAMBDA .005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99501	.99005	.98020	.97045	.96080
0.15	.99925	.99850	.99253	.98512	.97047	.95604	.94182
0.20	.99900	.99801	.99007	.98025	.96088	.94191	.92330
0.25	.99876	.99752	.98764	.97544	.95148	.92812	.90533
0.30	.99852	.99703	.98525	.97072	.94231	.91474	.88799
0.35	.99828	.99656	.98292	.96613	.93343	.90185	.87137
0.40	.99805	.99610	.98065	.96168	.92488	.88952	.85556
0.45	.99782	.99565	.97847	.95741	.91671	.87781	.84064
0.50	.99761	.99523	.97637	.95334	.90897	.86678	.82667
0.55	.99741	.99482	.97439	.94948	.90168	.85647	.81371
0.60	.99721	.99444	.97251	.94584	.89488	.84691	.80177
0.65	.99703	.99407	.97075	.94245	.88856	.83811	.79087
0.70	.99686	.99373	.96911	.93929	.88274	.83007	.78101
0.75	.99670	.99342	.96758	.93638	.87742	.82278	.77214
0.80	.99656	.99313	.96618	.93370	.87258	.81622	.76425
0.85	.99642	.99286	.96489	.93126	.86821	.81036	.75727
0.90	.99630	.99261	.96371	.92905	.86428	.80515	.75117
0.95	.99618	.99238	.96264	.92704	.86078	.80057	.74586
1.00	.99608	.99218	.96168	.92524	.85767	.79657	.74130
Z1	.99776	.99553	.97795	.95661	.91597	.87786	.84212
ZBAR	.99526	.99057	.95455	.91304	.84000	.77778	.72414

ALPHA= 4.0 PZERO= 0.70 MU= 4.76

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99501	.99005	.98020	.97045	.96080
0.15	.99925	.99850	.99254	.98513	.97049	.95606	.94184
0.20	.99900	.99801	.99009	.98027	.96093	.94198	.92340
0.25	.99876	.99752	.98767	.97550	.95161	.92831	.90558
0.30	.99852	.99705	.98532	.97086	.94258	.91513	.88851
0.35	.99829	.99658	.98304	.96637	.93390	.90256	.87229
0.40	.99807	.99614	.98085	.96208	.92565	.89066	.85705
0.45	.99785	.99571	.97876	.95800	.91787	.87952	.84287
0.50	.99765	.99531	.97680	.95418	.91061	.86919	.82982
0.55	.99746	.99494	.97496	.95061	.90389	.85971	.81791
0.60	.99729	.99459	.97325	.94731	.89773	.85108	.80718
0.65	.99713	.99426	.97168	.94429	.89213	.84330	.79759
0.70	.99698	.99396	.97025	.94153	.88708	.83636	.78911
0.75	.99684	.99369	.96894	.93904	.88256	.83021	.78169
0.80	.99672	.99345	.96776	.93681	.87855	.82481	.77526
0.85	.99661	.99323	.96671	.93482	.87501	.82012	.76974
0.90	.99651	.99303	.96576	.93305	.87192	.81608	.76506
0.95	.99642	.99285	.96493	.93150	.86924	.81263	.76113
1.00	.99634	.99269	.96419	.93013	.86692	.80970	.75787
Z1	.99784	.99568	.97870	.95807	.91877	.88191	.84730
ZBAR	.99582	.99167	.95968	.92248	.85612	.79866	.74843

ALPHA= 4.0 PZERO= 0.80 MU= 5.52

TIME	LAMBDA .005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99502	.99006	.98021	.97046	.96081
0.15	.99925	.99850	.99255	.98515	.97051	.95610	.94190
0.20	.99901	.99801	.99011	.98032	.96103	.94212	.92359
0.25	.99877	.99753	.98773	.97562	.95184	.92865	.90603
0.30	.99853	.99707	.98544	.97109	.94304	.91582	.88941
0.35	.99831	.99663	.98324	.96678	.93471	.90376	.87387
0.40	.99810	.99620	.98118	.96273	.92694	.89256	.85954
0.45	.99790	.99581	.97925	.95897	.91976	.88229	.84650
0.50	.99772	.99545	.97747	.95551	.91321	.87300	.83478
0.55	.99755	.99512	.97585	.95236	.90731	.86470	.82440
0.60	.99740	.99481	.97438	.94953	.90205	.85736	.81532
0.65	.99727	.99454	.97307	.94701	.89740	.85096	.80748
0.70	.99714	.99430	.97190	.94478	.89334	.84543	.80078
0.75	.99704	.99409	.97087	.94283	.88983	.84070	.79515
0.80	.99694	.99390	.96997	.94113	.88682	.83671	.79046
0.85	.99686	.99373	.96919	.93967	.88427	.83338	.78661
0.90	.99679	.99359	.96852	.93842	.88212	.83062	.78350
0.95	.99673	.99347	.96794	.93735	.88032	.82837	.78102
1.00	.99667	.99336	.96745	.93645	.87884	.82656	.77908
Z1	.99795	.99590	.97974	.96013	.92271	.88757	.85454
ZBAR	.99639	.99281	.96503	.93243	.87342	.82143	.77528

ALPHA= 4.0 PZERO= 0.90 MU= 6.70

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99502	.99006	.98022	.97048	.96083
0.15	.99925	.99851	.99256	.98518	.97058	.95619	.94202
0.20	.99901	.99802	.99016	.98042	.96124	.94243	.92400
0.25	.99878	.99756	.98786	.97587	.95233	.92938	.90700
0.30	.99856	.99712	.98568	.97157	.94400	.91724	.89128
0.35	.99835	.99671	.98366	.96760	.93633	.90615	.87701
0.40	.99816	.99633	.98181	.96399	.92940	.89620	.86430
0.45	.99799	.99599	.98015	.96075	.92325	.88743	.85320
0.50	.99784	.99569	.97868	.95789	.91787	.87983	.84367
0.55	.99771	.99543	.97739	.95541	.91323	.87335	.83563
0.60	.99760	.99520	.97628	.95327	.90930	.86791	.82896
0.65	.99750	.99500	.97533	.95146	.90599	.86341	.82353
0.70	.99741	.99483	.97453	.94993	.90326	.85975	.81917
0.75	.99734	.99469	.97385	.94867	.90103	.85680	.81574
0.80	.99728	.99457	.97329	.94763	.89922	.85448	.81308
0.85	.99723	.99448	.97283	.94678	.89778	.85266	.81107
0.90	.99719	.99440	.97246	.94609	.89665	.85127	.80958
0.95	.99716	.99433	.97215	.94554	.89576	.85022	.80850
1.00	.99713	.99428	.97191	.94510	.89508	.84944	.80775
Z1	.99811	.99623	.98136	.96328	.92875	.89625	.86563
ZBAR	.99702	.99407	.97101	.94366	.89333	.84810	.80723

ALPHA= 4.0 PZERO= 0.95 MU= 7.80

	LAMBDA						
TIME	.005	.010	.050	.100	.200	.300	.400
0.05	.99975	.99950	.99750	.99501	.99005	.98511	.98020
0.10	.99950	.99900	.99502	.99007	.98024	.97050	.96086
0.15	.99926	.99851	.99258	.98522	.97066	.95632	.94219
0.20	.99902	.99804	.99023	.98056	.96150	.94283	.92453
0.25	.99879	.99759	.98801	.97618	.95294	.93029	.90820
0.30	.99859	.99718	.98597	.97215	.94513	.91892	.89350
0.35	.99840	.99680	.98413	.96854	.93817	.90887	.88058
0.40	.99823	.99647	.98251	.96537	.93211	.90018	.86952
0.45	.99809	.99619	.98110	.96263	.92694	.89283	.86025
0.50	.99797	.99594	.97991	.96032	.92260	.88675	.85267
0.55	.99786	.99573	.97891	.95840	.91904	.88181	.84659
0.60	.99778	.99556	.97808	.95682	.91616	.87788	.84182
0.65	.99771	.99542	.97741	.95554	.91387	.87480	.83816
0.70	.99765	.99531	.97687	.95452	.91207	.87244	.83541
0.75	.99760	.99522	.97644	.95372	.91069	.87065	.83339
0.80	.99757	.99515	.97610	.95309	.90963	.86934	.83195
0.85	.99754	.99509	.97583	.95261	.90884	.86838	.83095
0.90	.99752	.99504	.97563	.95224	.90826	.86771	.83028
0.95	.99750	.99501	.97547	.95196	.90783	.86724	.82985
1.00	.99749	.99498	.97535	.95175	.90753	.86693	.82960
Z1	.99825	.99651	.98277	.96604	.93403	.90382	.87528
ZBAR	.99744	.99490	.97500	.95122	.90698	.86657	.82979

APPENDIX B

EXAMPLE PROBLEM

Suppose a system consisting of 15 components is connected in series. Each component, operating independently of the others, has an exponential failure density with parameter λ and a special Erlang repair density with parameters μ and α. Assume that uptimes and downtimes are all independent.

a. What is the average availability of the system over the first mission time if;

(1) $\lambda = .005$ failures per mission time, $\mu = 4.8$ repairs per mission time, and $\alpha = 2$?

(2) $\lambda = .40$ failures per mission time, $\mu = 4.2$ repairs per mission time, and $\alpha = 4$?

b. What is the long-run average availability of the system if;

(1) the parameters are the same as in a(1)?

(2) the parameters are the same as in a(2)?

Solution:

a.

(1) The average availability of a single component over the first mission time is $Z1 = .99856$ (from Appendix A). Letting η_s be the average system availability over the first mission time;

$$\eta_s = (Z1)^{15} = (.99856)^{15} \doteq \underline{\underline{.9786}}\ .$$

(2) In this case Z1 = .84212 and $\eta_s = (.84212)^{15} \doteq \underline{\underline{.0760}}$.

b.

(1) The long-run average availability of a single component is ZBAR = .99792.

Letting $\bar{\eta}_s$ be the long-run average availability of the system,

$$\bar{\eta}_s = (\text{ZBAR})^{15} = (.99792)^{15} \doteq .\underline{\underline{9693}} \; .$$

This compares with the average system availability over the first mission time of .9786.

(2) In this case ZBAR = .72414 and $\bar{\eta}_s \doteq .\underline{\underline{0079}}$.

This compares with the average system availability over the first mission time of .076.

The above comparisons indicate the use of the long-run average availability to approximate the system availability over the first mission time is a conservative practice. These cases and others also indicate that the lower the value of ZBAR the more conservative is the practice.

APPENDIX C

FORTRAN IV COMPUTER PROGRAM FOR THE INVERSION OF $\eta^*(s)$ BY NUMERICAL METHODS

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C     THIS PROGRAM COMPUTES VALUES OF SYSTEM AVAILABILITY BY
C   INVERTING THE LAPLACE TRANSFORM OF THE AVAILABILITY
C   FUNCTION.
C
        IMPLICIT REAL*8(A-H),REAL*8(O-Z)
        COMPLEX*16 S,ETA,EXPON,CONST,TOP,BOTTOM
        DIMENSION XCOF(20),COF(20),ROOTR(20),ROOTI(20),S(20),
       1EXPON(20),RETA(20,7),YLAMDA(7),ZBAR(7),Z1(7),ETA(20)
        DATA YLAMDA/0.005,0.010,0.05,0.1,0.2,0.3,0.4/
        JCASE=0
C
C   ICASE IS THE NUMBER OF CASES TO BE PROCESSED THIS RUN.
        READ(5,1)ICASE
      1 FORMAT(I2)
      5 READ(5,10)ALPHA,XMU,PZERO
     10 FORMAT(F3.0,2F10.3)
        JCASE=JCASE+1
C
        WRITE(6,6)JCASE
      6 FORMAT(1H1,45X,'CASE',I4)
        DO 800 K=1,7
        XLAMDA=YLAMDA(K)
C
        WRITE(6,7)
      7 FORMAT(//10X,'INPUT DATA')
        WRITE(6,8)ALPHA, XMU,XLAMDA
      8 FORMAT(/10X,'ALPHA = ',F3.0,5X,'    MU =',F10.3,5X,
       1'LAMBDA =',F10.3)
C
C   PUT ZEROS IN ALL ARRAYS.
        DO 80 L=1,20
        XCOF(L)=0.0
        COF(L)=0.0
        ROOTR(L)=0.0
        ROOTI(L)=0.0
        S(L)=(0.0,0.0)
        ETA(L)=(0.0,0.0)
     80 EXPON(L)=(0.0,0.0)
C
        M=ALPHA
        N=M+1
        IF(M.NE.ALPHA)GO TO 40
C
C   COMPUTE COEFFICIENTS OF THE POLYNOMIAL.
        CALL COEFF(ALPHA,XMU,XLAMDA,XCOF)
C
C   FIND THE ROOTS OF THE POLYNOMIAL.
C   DPOLRT IS A PROGRAM LIBRARY SUBROUTINE.
        CALL DPOLRT(XCOF,COF,N,ROOTR,ROOTI,IER)
C
     60 WRITE(6,61)IER
C   IER IS AN ERROR CODE FROM SUBROUTINE DEPOLRT.
     61 FORMAT(//10X,'IER=',I2)
        WRITE(6,18)
     18 FORMAT(/13X,'ROOTS OF THE POLYNOMIAL.')
        WRITE(6,19)
     19 FORMAT(/15X,'REAL',6X,'IMAGINARY'/ )
     14 WRITE(6,16)(ROOTR(I),ROOTI(I),I=1,N)
     16 FORMAT(10X,2F12.5)
C
C   CHECK FOR MULTIPLE ROOTS.
        CALL CKMULT(ROOTR,ROOTI,M)
```

```
C
C   PUT THE ROOTS INTO THE COMPLEX ARRAY S(20).
      DO 100 I=1,N
  100 S(I)=DCMPLX(ROOTR(I),ROOTI(I))
C
C   BEGIN INVERSION BY HEAVISIDE'S EXPANSION FORMULA.
C
      DO 150 I=1,N
      TOP=XMU+S(I)
      BOTTOM=TOP+M*(XLAMDA+S(I))
      CONST=TOP/BOTTOM
      DO 150 J=1,20
      EXPON(J)=CONST*CDEXP(J*S(I)*0.05)
  150 ETA(J)=ETA(J)+EXPON(J)
C
      WRITE(6,160)
  160 FORMAT(///10X,'VALUES OF THE AVAILABILITY FUNCTION AT'
     1' .05 MISSION-TIME INCREMENTS.'/)
      DO 170 I=1,20
      T=0.05*I
  170 WRITE(6,171)T,ETA(I)
  171 FORMAT(10X,F4.2,2F10.6)
C
      SUM=0.0
      DO 155 L=1,20
      RETA(L,K)=REAL(ETA(L))
  155 SUM=SUM+RETA(L,K)
      TOTSUM=SUM+SUM+1.-RETA(20,K)
C
C   Z1 IS THE AVERAGE AVAILABILITY OVER THE FIRST MISSION-TIM
      Z1(K)=TOTSUM/40.
C
C   ZBAR IS THE LONG RUN TIME AVERAGE AVAILABILITY.
  900 ZBAR(K)=XMU/(ALPHA*XLAMDA+XMU)
C
      CALL OUTPUT(YLAMDA,ALPHA,PZERO,XMU,Z1,ZBAR,RETA)
C
   30 IF(JCASE-ICASE)5,50,50
   40 WRITE(6,41)
   41 FORMAT(10X,'ERROR- ALPHA MUST BE AN INTEGER')
      GO TO 30
   50 STOP
      END


      SUBROUTINE CKMULT(ROOTR,ROOTI,M)
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION S(1),ROOTR(1),ROOTI(1)
C
C   THIS SUBROUTINE CHECKS FOR MULTIPLICITIES AMONG THE ROOTS
C
      MULT=1
      DO 50 I=1,M
      K=M+1-I
      DO 50 J=1,K
   50 IF(ROOTR(I).EQ.ROOTR(I+J).AND.ROOTI(I).EQ.ROOTI(I+J))
     1MULT=MULT+1
      IF(MULT.NE.1) GO TO 60
C
      WRITE(6,51)
   51 FORMAT(///10X,'THERE ARE NO MULTIPLE ROOTS.')
      RETURN
C
   60 WRITE(6,61)
   61 FORMAT(///10X,'THE POLYNOMIAL HAS MULTIPLE ROOTS. THIS
     1'PROGRAM WILL NOT ACCURATELY HANDLE THIS CASE.')
      RETURN
      END
```

```
      SUBROUTINE COEFF(ALPHA,XMU,XLAMDA,XCOF)
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION XCOF(1)
C
C  THIS SUBROUTINE COMPUTES THE COEFFICIENTS OF THE EXPANDED
C  POLYNOMIAL IN THE DENOMINATOR OF THE LAPLACE TRANSFORM.
C
      M=ALPHA
      N2=M+2
      X=ALPHA+1.0
      XCOF(1)=0.0
      XCOF(M+2)=1.0
      CHOOZ=M
      DO 20 I=1,M
      BKT=XLAMDA+(XMU*I)/(X-I)
      XCOF(I+1)=CHOOZ*BKT*XMU**(M-I)
   20 CHOOZ=CHOOZ*(M-I)/(I+1)
C
C  THE COEFFICIENTS ARE NOW IN THE ARRAY,XCOF.
C
      WRITE(6,17)
   17 FORMAT(//10X,'COEFFICIENTS OF THE POLYNOMIAL')
      WRITE(6,12)(XCOF(I),I=1,N2)
   12 FORMAT( /10X,10F12.6)
      RETURN
      END


      SUBROUTINE OUTPUT(YLAMDA,ALPHA,PZERO,XMU,Z1,ZBAR,RETA)
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION YLAMDA(1),Z1(1),ZBAR(1),RETA(20,7)
      WRITE(6,1)ALPHA,PZERO,XMU
    1 FORMAT(1H1,///////25X,'ALPHA=',F4.1,5X,'PZERO=',F5.2,
     15X,'MU=',F5.2)
      WRITE(6,11)
   11 FORMAT(//46X,'LAMBDA')
      WRITE(6,2)(YLAMDA(L),L=1,7)
    2 FORMAT(/22X,F4.3,6(4X,F4.3))
      WRITE(6,12)
   12 FORMAT(15X,'TIME'/)
      DO 50 I=1,20
      T=0.05*I
   50 WRITE(6,3)T,(RETA(I,J),J=1,7)
    3 FORMAT(15X,F4.2,7(2X,F6.5)/)
C
      WRITE(6,4)(Z1(I),I=1,7)
    4 FORMAT(/16X,'Z1',1X,7(2X,F6.5)//)
      WRITE(6,5)(ZBAR(I),I=1,7)
    5 FORMAT(15X,'ZBAR',7(2X,F6.5))
      RETURN
      END
```

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

INVESTIGATION OF THE ACCURACY OF USING STEADY-STATE RESULTS TO APPROXIMATE ACTUAL SYSTEM AVAILABILITY

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's thesis; April 1970

5. AUTHOR(S) (First name, middle initial, last name)

George Richard Catron, Major, United States Army

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April, 1970	41	5

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Values of system availability are computed for a system whose failure density is exponential (λ) and whose repair density is special Erlang (μ,α). The system is modeled as an alternating renewal process. The Laplace transform of the availability function is developed and inverted (by using a numerical procedure) to obtain the values of the system availability.

Corresponding values of the long-run average system availability and the average availability over the first mission time are also computed. Comparisons are made which establish that using the long-run average value to approximate the true availability over the first mission time is a conservative practice.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SYSTEM AVAILABILITY						
ALTERNATING RENEWAL PROCESS						
INVERSE LAPLACE TRANSFORM						

Thesis 118435
C333 Catron
c.1 Investigation of
the accuracy of using
steady-state results
to approximate actual
system availability.